Engineering Notes

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Extension of Region of Convergence in Orbit Determination Using Function-Minimization Techniques

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Introduction

THE ability to determine accurately the state of an orbiting satellite from ground observations is essential to meet several operational and nonoperational objectives. The state of the satellite center of gravity at any given time can be defined either by the six orbital elements (a, e, i, ω, Ω, and E) or by a state vector giving the position and velocity components. The state is determined from noisy ground observation data. The observations usually provide information on range, range rate, elevation, and azimuth of the satellite from the ground station.

The differential correction methods, widely used in the orbit determination problem, have been well known since the time of Gauss. This linear least-squares procedure has proved itself to be a simple but useful means of providing orbit estimation. Innumerable operational computer packages for orbit determination employ the differential correction method.[1,2]

However, it has been noticed by several workers[3-5] that the classical differential correction method (CDCM) is limited in its applicability. Orbit determination methods based on differential corrections require an initial state-vector estimate sufficiently close to the desired solution for the iteration process to converge to the solution. This constraint on the convergence region of the initial guesses can be a consequence of the linearization of the nonlinear dynamical equations of motion of the satellite. Sometimes numerical inaccuracies also can contribute to the divergence of the differential correction procedure.

Several attempts have been made to improve the convergence properties of the methods in orbit determination. Tang and Greer[3] used certain modifications of the basic CDCM. Hunt and Nayfeh[5] have evolved a nonlinear correction procedure using the fact that if the observation noise is random, then the least-square fit to the residuals will be the time axis itself. Marquardt[6] has proposed a modification to CDCM so that numerical inaccuracies in matrix inversion can be controlled. Greenstadt[7] gives another elegant method to avoid inversion-divergence. Popham[8] uses a quasilinearization method for the estimation of geophysical constants from satellite observations, while Piggott[9] uses minimization methods to the same problem. Bard[10] makes a general study of the use of gradient minimization methods for the solution of some nonlinear parameter estimation problems.

The present study is motivated by the opportunity to utilize the recent progress in the field of nonlinear optimization in orbit determination. Initial studies[11] have shown that the convergence characteristics of some of the optimization methods are far superior to those of the differential correction and related methods. In this Note, we present typical results to substantiate the expansion of the convergence regions when nonlinear methods are used.

Optimization Methodology

Many of the nonsequential estimation procedures can be treated as straightforward function-minimization problems.[10,11] The maximum likelihood and weighted least-squares estimators, commonly employed in orbit determination, fall into this category. In the present study, the maximum likelihood method is used, under the assumptions that the time of observations is precisely known and that the observations made at each instant of time are subjected to normally distributed and uncorrelated errors with zero mean and known variances.

While solving the minimization problem, various difficulties can arise due to the nonlinear behavior of the functions to be minimized. Nonlinear optimization techniques currently studied in the literature can be divided into two broad groups – either gradient-dependent or gradient-independent. In both these groups, some methods require a linear minimization in the space of parameters to be estimated. The nonlinear techniques are usually generated with the aim of minimizing a quadratic function of p variables in a finite number (usually p) of iterations, a property known as quadratic convergence.

In the present study, we apply the best known methods of the aforementioned two groups to the problem of orbit determination, with the hope of finding algorithms that do not suffer from the limitations of the classical differential correction techniques. The choice of the optimization methods is based on their reported performance on various test functions. The following methods are employed: 1) Davies, Swann and Campey method (DSC); 2) Brent's modification of Powell's conjugate direction method (Brent); 3) simplex evolutatinary operation method, as improved by Parkinson and Hutchinson (PHS); 4) conjugate gradient method (CG); 5) Davidon-Fletcher-Powell-Stewart variable metric method (Stewart); 6) Fletcher's dual variable metric algorithm (Fletcher); 7) differential correction with Greenstadt's technique (DCG); and 8) differential correction with Greenstadt's technique and linear search (GLS).

In addition, the results for classical differential correction method (CDCM) are also obtained for comparison purposes. A discussion of all these methods, various modifications introduced, and the salient features of the one-dimensional-search method employed are included in Ref. 11, where the original reference can be found. A general perturbation type of dynamic model is employed to describe the motion of the satellite. As the purpose of this study is to prove the applicability and utility of nonlinear optimization techniques for orbit determination, no attempt is made to use a very accurate dynamic model. For example, oblateness effects are included only to fourth order; seasonal and other variations in the atmospheric model are not taken into account; and solar radiation and luni-solar perturbations are all neglected.

Discussion of the Results

Observation data from a down-range station are simulated for the first three passes of the proposed Indian satellite RS-1.

Received Feb. 8, 1978.
Index categories: Earth-Orbital Trajectories; Spacecraft Signatures and Tracking; Analytical and Numerical Methods.
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Table 1 Initial guess values

Case no.	a, km	e	i, deg	ω, deg	Ω, deg	E deg
1	6983.0	.044	40.27	172.1	275.78	0.0
2	6980.0	.042	40.0	172.0	275.0	0.0
3	6975.0	.04	40.0	171.0	275.0	0.0
4	6950.0	.025	35.0	172.0	265.0	3.0
5	6900.0	.025	30.0	170.0	260.0	10.0
Nominal	6983.21	.0437	40.27	172.13	275.777	0.0

Table 2 Typical performance of various optimization methods for orbit determination

Method	Case 1	Case 2	Case 3	Case 4	Case 5
DSC	C (16)[a]	F[b]	F	F	F
Brent	C (10)	C (18)	C (20)	F	F
Powell	C (7)	C (14)	C (18)	F	F
PHS	C (20)	F	F	F	F
CG	C (2)	F	F	F	F
Stewart	C (6)	C (4)	C (4)	C (19)	C (16)
Fletcher	C (6)	C (4)	C (4)	C (12)	C (14)
DCG	C (2)	C (2)	C (2)	F	F
GLS	C (1)	C (1)	C (4)	C (12)	F
CDCM	C (2)	C (2)	C (2)	F	F

[a] C = convergence (time taken for convergence is given, in parentheses, in minutes on an IBM 360 computer). [b] F = failure to converge.

Table 3 Range of initial guesses for which convergence is obtained

Element	CDCM	GLS	Stewart/ Fletcher
a, km	(6983, 6983.4)	(6981, 6985)	(6720, 7350)
e	(.04, .0475)	(.015, .07)	(0, 0.45)
i, deg	(40, 40.5)	(25, 50)	(0, 180)
ω, deg	(171.9, 172,4)	(171.5, 172.8)	(0, 360)
Ω, deg	(275.6, 275.9)	(275, 277)	(0, 360)
E, deg	$0 \pm .25$	$0 \pm .8$	0 ± 180

Gaussian noise is added to these analytical values before subjecting them to processing. Details of these simulations are given in Ref. 11. Table 1 gives five sets of values of orbital elements which are used as initial guesses. A typical performance of various methods for single-pass data is shown in Table 2.

Table 2 shows that the performance of the gradient-independent methods (DSC, Brent, Powell, and PHS) is inferior to that of the gradient-dependent methods. Most of the methods converge in cases 1-3, where the initial values are comparatively nearer to the actual orbital elements. The classical differential correction method (CDCM) and the two methods based thereon (DCG, GLS) failed when the initial values are far away from the actual elements. However, when successful, these methods converge quickly. Of these three methods, the Greenstadt correction with linear search (GLS) performed best. The Stewart and Fletcher methods turn out to be the best among all in the sense that they converged even when the initial conditions are far off. Fletcher's dual variable metric method is the more preferable of the two, since it takes less time.

A study is made to find the convergence range of Fletcher's method. A single element is varied while the remaining five are taken with their actual values. Comparison with classical differential correction method is given in Table 3 for a typical analysis of three-pass data.

These studies show that, for the problem of orbit estimation, variable metric and related optimizati^n methods present much better convergence characteristics than the differential correction methods. The extension in the convergence region by using nonlinear optimization methods enhances the success of obtaining a better estimate of state vector in a shorter tracking period.

Acknowledgment

The authors wish to thank C.L. Amba-Rao and N.S. Venkataraman for their encouragement.

References

[1] Gooding, R.H., "Orbit Determination at the Royal Aircraft Establishment," RAE – Tech. Memo Space-156, 1970.

[2] Ball, J.E., "Preliminary Circular Orbit from a Single Station of Range-only Data," *AIAA Journal,* Vol. 5, Dec. 1967, pp. 2261-2264.

[3] Tang, C.C.H. and Greer, C.L., "Determination of Orbits of Planetary Gravitational Fields," *AIAA Journal,* Vol. 7, Aug. 1969, pp. 1469-1476.

[4] Rourke, K.H. and Jordan, J.F., "Application of Sequential Filtering to Estimation of the Inter-Planetary Orbit of Mariner 9," *Journal of Spacecraft and Rockets,* Vol. 10, Dec. 1973, pp. 773-778.

[5] Hunt, W.E. and Nayfeh, A.H., "A Nonlinear Correction Method for Orbit Determination," *AIAA Journal,* Vol. 5, June 1967, pp. 1183-1185.

[6] Marquardt, D.W., "An Algorithm for Least Squares Estimation of Nonlinear Parameters," *SIAM Journal of Numerical Analysis,* Vol. 11, 1963, pp. 431-441.

[7] Greenstadt, J., "On Relative Efficiencies of the Gradient Methods," *Mathematics of Computation,* Vol. 21, July 1967, pp. 360-367.

[8] Popham, S., "An Application of the Method of Quasi-linearization to a Problem of Nonlinear Curve-Fitting Arising in the Estimation of Geophysical Constant from Satellite Observations," RAE-TR No. 67085, 1967.

[9] Piggot, B.A.M., "An Application of the Methods of Function Minimisation to a Problem of Non-linear Curve-Fitting Arising in the Estimation of Geophysical Constants from Satellite Observations," RAE-TR No. 66260, 1966.

[10] Bard, Y., "Comparison of Gradient Methods for the Solution of Nonlinear Estimation Problems," *SIAM Journal of Numerical Analysis,* Vol. 7, 1970, pp. 157-185.

[11] Adimurthy, V. and Joy, K.V., "Estimation of Orbital Elements using Methods of Nonlinear Optimization," ISRO-VSSC-SN-02-77, Indian Space Research Organization, Bangalore, 1977, also, presented at the 12th International Symposium on Space Technology and Science, Tokyo, May 1977.

Stability of Spacecraft During Asymmetrical Deployment of Appendages

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Nomenclature

$a_1(t), a_2(t)$ = time-varying terms in the differential equations for h_1 and h_2
c = boom extension rate
h_1, h_2, h_3 = components of the angular momentum vector along principal axes

Received May 4, 1978; revision received Aug. 4, 1978.

Index category: Spacecraft Attitude Dynamics and Control.

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